



Numeracy: Analysis of Representation Translation Errors

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Abstract

This research aims to describe students' representational translation errors in solving numeracy problems. A qualitative method was employed in this study. The research subjects consisted of 24 elementary school students from a school in Malang Regency. The instruments used included tests, observation sheets, recording equipment, and field notes. The research process involved recording students' answers, identifying students who completed their work correctly but could not explain their answers, and categorizing students who made errors based on the type of error. The findings indicate that students experience translation errors in language representations, symbols, pictures, and analogies. The study recommends that teachers implement targeted learning interventions to support students who continue to make such errors.

Keywords: errors; numeracy; representation; translation

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INTRODUCTION

Many experts define the word numeration. Numeracy is a student's basic ability when applying number concepts and operations in everyday life (Setiawan & Sukanto, 2021). Numeracy is not just counting but the interpretation of calculations or understanding the relationship between numbers (Kus, 2018). Numeracy can explain graphs, tables and mathematical problems (Situmorang et al., 2023). Numeracy is the ability to recognize, interpret, determine patterns and relationships in solving mathematical problems (Subekti et al., 2022). Numeracy is the ability to interpret mathematical ideas related to numbers (Yustitia et al., 2021).

Numeracy related to numbers is needed to solve everyday problems. Someone who has good numeracy skills will be able to analyze and conclude problems well (Xiao et al., 2019). The numeracy problem that is often encountered is that students have difficulty applying the concept of whole numbers in everyday life (Alan & Akbaş, 2023).

Several experts have researched students' errors regarding integers in elementary school. Students experience errors comparing and subtracting whole numbers (Bozkurt et al., 2022). Giving routine questions causes students to make many mistakes in solving complex problems on integers (Rohmah et al., 2022). Most students make mistakes when

representing subtraction of integers (Sari et al., 2020).

Some researchers also analyze representation translation errors in numeracy problems. Students still have difficulty representing symbols, language and pictures when solving number sense problems (Murniasih et al., 2018; Arifin et al., 2024). Students are less fluent when solving number representation problems when calculating the slope of a tangent line (Arefaine et al., 2022). Students have difficulty constructing triangular pictures representations due to poor understanding of the prerequisite material (Mhlolo, 2015). Based on the explanation above, researchers are interested in examining numeration problems in the translation of representations of integer operations.

There are several theories of representational translation that have been used by many researchers. Johnson's model uses translation of pictures representation, manipulation, symbols, language, technology and real life situations (Johnson, 2018). Lesh's model uses translation of representations of symbols, pictures, manipulation, real life situations and language (Abed & Hassan, 2021). There are several domains of representation including verbal, pictorial, algebraic, and number (Mainali, 2021). This research adapts Lesh's model, namely representation: symbols, language and pictures. The Lesh model is used because it can be used to analyze the translation of number numeracy representations (Murniasih et al., 2018). This research aims to describe translation errors in the representation of numeracy problems in integer operations. Research analyzing translation errors in the representation of numeracy problems in students is important so that teachers can provide appropriate treatment so that similar errors can be minimized.

RESEARCH METHODOLOGY

This research aims to reveal translation errors in students' representations of numeracy problems. Representation translation errors are focused on problems of symbols, language and pictures. In particular, this research explains the process of changing from one representation to another. An adaptation of the Lesh model (Abed & Hassan, 2021) to analyze the representation translation process can be seen in Figure 1 below.

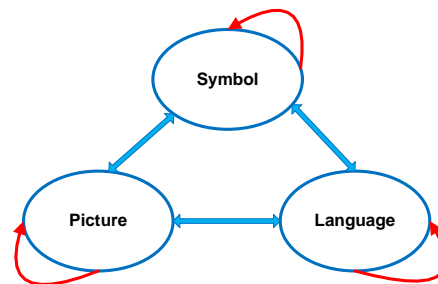


Figure 1. Adaptation of Lesh Model Representation Translation (Abed & Hassan, 2021)

Qualitative data is used in this research, because it can interpret, explain and classify the data obtained (Murniasih et al., 2020). The type of research is descriptive exploratory because the researcher wants to obtain detailed data naturally related to the translation of numeracy representations of numeracy problems in students (Stewart et al., 2021).

The research subjects were 24 elementary school students at one of the schools in Malang Regency. All students are given two test question numbers related to numeracy. The results of students' solutions are corrected and then those that are still incorrect are grouped based on the type of translation representation of the error. Meanwhile, students' correct answers are not analyzed. The steps for selecting research subjects can be seen in Figure 2 below.

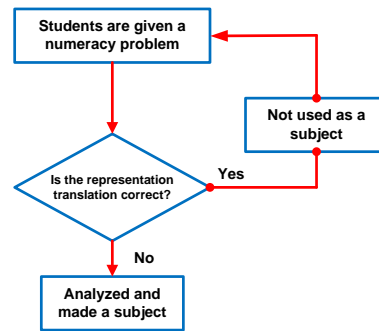


Figure 2. Selection of Research Subjects

This research uses instruments in the form of test sheets, interview guides, field notes, and recording tools. The test sheet is used to determine the translation process of students' representations in working on numeracy problems. The test question sheet for students can be seen in Figure 3.

1. Letakkan angka satu sampai duabelas (angka hanya dapat digunakan satu kali) pada lingkaran yang kosong! Sehingga jumlah setiap sisi segitiga adalah tiga puluh enam.

2. Dua bentuk di bawah ini masing-masing mempunyai nilai bilangan bulat antara 1 sampai 10.

Tuliskan nilai bentuk berikut ini!

Setelah mengetahui nilainya kerjakan soal berikut ini!

- $\text{pentagon} + \text{triangle} = \underline{\hspace{2cm}}$
- $\text{triangle} + \text{pentagon} \times \text{triangle} = \underline{\hspace{2cm}}$
- $\text{triangle} + \text{pentagon} \times \text{pentagon} = \underline{\hspace{2cm}}$
- $\text{triangle} + \text{pentagon} \times \text{triangle} - \text{pentagon} = \underline{\hspace{2cm}}$
- $\text{triangle} \times \text{triangle} - \text{pentagon} = \underline{\hspace{2cm}}$

Figure 3. Test Sheet (Adapted from Meg et al., (2003))

Recording equipment is used to record students when working on questions and interviews. An interview guide was used to reveal more deeply students' errors related to translation of representations of numeracy problems. Field notes are used to record student movements when solving test questions.

The data analysis used by researchers was adapted from the Lesh Model (Abed & Hassan, 2021). The instrument was validated by 2 experts, namely a mathematician (V1) and a learning expert (V2). The instrument validation criteria in Table 1. are adapted from research by Murniasih et al. (2018).

Table 1. Validation Result Criteria Murniasih et al., (2018)

Standard Criteria	Category
86% - 100%	Very valid
70% - 85%	Valid
60% - 69%	Not valid
0% - 59%	Very not valid

RESULTS AND DISCUSSION

The instrument validation results show an average score of 88% or in other words in the very valid category. The assessment results from validators 1 and 2 can be seen in Table 2 below.

Table 2. Validator Assessment Results

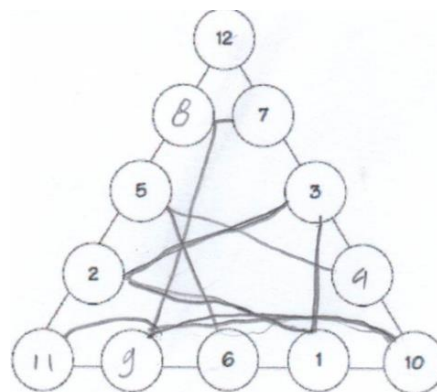
Validator	Assessment Percentage
V1	85%
V2	91%
Average	88%
Category	Very valid

The results of student work are then grouped based on the type of error. The classification of error types based on student work can be seen in Table 3.

Table 3. Types of Representational Translation Errors

Test questions	Error Type		
	Type 1	Type 2	Type 3
Number 1	The sum of the numbers on two sides of the triangle is not 36	The sum of the numbers on one side of the triangle is not 36	The same number is used more than once
Number 2	Cannot translation representation from picture to symbol	Incorrectly completing the sequence of operations	Cannot distinguish number operations

Based on the results of the students' work, it was found that there were 3 students as interview subjects for tests number one and two. Student errors in working on test question number 1 are classified into 3 types. Type 1 errors were made by 5 students, namely: S1 (Subject 1), S7, S11, S16, and S17. S1 was chosen as the interview subject because he was communicative when interviewed (Murniasih et al., 2020). Type 1 representation translation errors can be seen in Figure 4.

**Figure 4. Type 1 Error by S1**

In Figure 4, S1 made errors in representing symbols, images and language in calculating two sides of a triangle. Based on the results of the interview with S1, S1 said that S1 understood that all the sides of a triangle must add up to 36. Next, S1 recalculated the right side of the triangle by adding $12 + 7 + 3 + 4 + 10 = 36$. Then S1 guesses that there are still numbers 8, 9, and 11 that have not been entered on the empty side of the triangle. S1 enters the three numbers on the bottom and left sides of the triangle which are still empty. Then the researcher asked S1 to count the number of numbers on the bottom side of the triangle $11 + 9 + 6 + 1 + 10 = 37$ and the left side of the triangle $11 + 2 + 5 + 8 + 12 = 37$. The results of the interview showed that S1 experienced translation errors in symbol and language representation. These results are in line with research by Smith et al. (2023) which states that understanding mathematics related to language representation is something complicated because it combines symbolic and pictorial representations. Apart from that, S1 is also less careful in calculations. Previous research says that many elementary school students lack skills in calculations (Desoete & Baten, 2022).

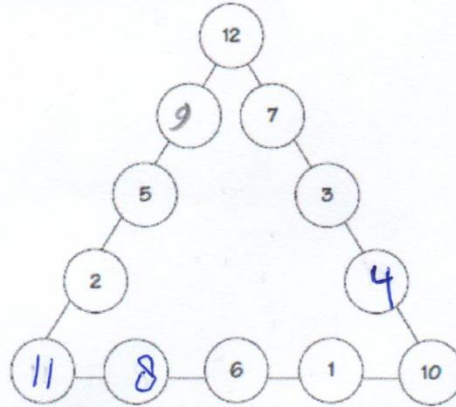


Figure 5. Type 2 Error by S8

Type 2 errors were made by 6 students, namely S3, S8, S15, S21, S22, and S24 in calculating one side of a triangle. Next, S8 was chosen as the interview subject. The S8 misrepresents language, symbols, and images (Figure 5.). Based on the results of the interview, S8 calculated the bottom and right sides of the triangle each totaling 36. However, when calculating the left side, S8 then added $11 + 2 + 5 + 9 + 12 = 39$. S8 just realized its mistake because it didn't double check the answer. This result is in line with research by Liew et al. (2022) which stated that students failed to conclude the final answer correctly because they did not check their answer again.

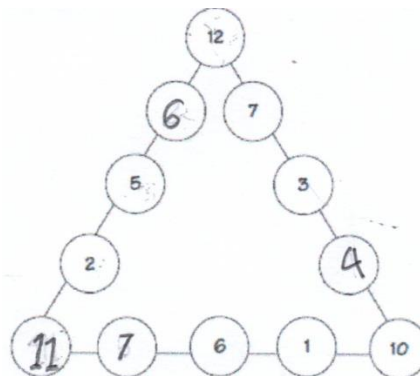


Figure 6. Type 3 Error by S13

There were 4 students who made type 3 errors, namely S2, S4, S13, and S18. Next, S13 was chosen as the interview subject. Based on S13's answer, he made errors in representing images, language and symbols (Figure 6.). This can be seen from S13's interview answer. Based on the addition of the bottom and left sides of the triangle, the results were $11 + 7 + 6 + 1 + 10 = 35$ and $11 + 2 + 5 + 6 + 12 = 36$, respectively. The sum on the left side is correct and the result is 36, but there is a number 6 that is used twice, while in the question it is said that the number 6 can only be used once. These results are in line with the opinion of Viseu et al. (2021) who said that students have difficulty with image, symbol and language representations when solving number calculation operation problems.

Students' mistakes in doing test question number 2 are also classified into three types. Type 1 errors were made by 7 students, namely S1, S4, S7, S16, S19, S21, and S22, then S1 was appointed as the interview subject (Figure 7.).

a. $\triangle + \triangle = 8$

b. $\triangle + \text{Pentagon} \times \triangle = 24$

c. $\triangle + \text{Pentagon} \times \text{Pentagon} = 43$

d. $\triangle + \text{Pentagon} \times \triangle - \text{Pentagon} = 27$

e. $\triangle \times \triangle - \text{Pentagon} = 4$

Figure 7. Type 1 Error by S1

S1 experiences errors in representing language, symbols, images and analogies. S1 did not understand that the pentagon image was analogous to the number 9 and the triangle image was analogous to the number 10. The interview results showed that S1 also misrepresented the language in the question (Figure 7.). Apart from that, S1 also does not understand the symbols for addition, multiplication and subtraction. The translation of analogical representation is a research finding because it is different from Lesh's model. Based on students' answers there is an analogous representation. The research results of Hicks (2020) say that analogies play an important role in the connection of images and calculations.

Type 2 errors in question number 2 were made by 5 students, namely S2, S5, S8, S17, and S24. Next, S8 was appointed as the interview subject (Figure 8.). S8 experiences translation errors in the representation of addition, subtraction and multiplication symbols.

b. $\triangle + \text{Pentagon} \times \triangle = 190$

c. $\triangle + \text{Pentagon} \times \text{Pentagon} = 171$

d. $\triangle + \text{Pentagon} \times \triangle - \text{Pentagon} = 181$

e. $\triangle \times \triangle - \text{Pentagon} = 91$

Figure 8. Type 2 Error by S8

In Figure 8, S8 computes operations from left to right. S8 does not prioritize the multiplication operation before addition or subtraction operations (Aydin-Guc & Aygun, 2021). So for question number 2 only part a is correct so it is not analyzed. Meanwhile, part b then S8 calculates $10 + 9 \times 10 = 190$. The correct answer should be $9 \times 10 + 10 = 100$. Part c then S8 calculates $(10 + 9) \times 9 = 171$. The correct answer for part c should be $10 + (9 \times 9) = 91$. Part d then S8 calculates $10 + 9 \times 10 - 9$ from addition, multiplication and subtraction operations. The correct answer should be $10 + (9 \times 10) - 9 = 91$. Meanwhile, for part e, the answer is correct.

a. $\text{Pentagon} + \text{Triangle} = 19$

b. $\text{Triangle} + \text{Pentagon} \times \text{Triangle} = 29$

c. $\text{Triangle} + \text{Pentagon} \times \text{Pentagon} = 29$

d. $\text{Triangle} + \text{Pentagon} \times \text{Triangle} - \text{Pentagon} = 38$

e. $\text{Triangle} \times \text{Triangle} - \text{Pentagon} = 29$

Figure 9. Type 3 Error by S13

There was one person (S13) who experienced a type 3 error. S13 experienced a translation error in symbol representation. S13, based on the results of the interview, added all the calculations. S13 does not see multiplication or subtraction symbols. So for part b S13 do the calculation $10 + 9 + 10 = 29$. Meanwhile, for part d, S13 performs the calculation $10 + 9 + 10 + 9 = 38$. Likewise for parts c and d then S13 adds up all the parts. Students often have difficulty representing arithmetic operations because they have not mastered the prerequisite material well (Rum & Juandi, 2022). Apart from that, a lack of understanding of concepts also causes errors (Rochma et al., 2023).

CONCLUSION AND SUGGESTIONS

Based on the results and discussion, it shows that students experience representational translation errors when taking tests which are classified into three types. A total of 15 students made translation errors in the representation for test number 1 and 13 students made translation errors in the representation for test number 2. According to Lesh's model, the errors that emerged were in the representation of language, symbols and images. Research findings show that another representation error namely analogical representation. In further research, it is hoped that teachers can provide learning interventions for students who still make representational translation errors.

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