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WHEN PROPORTION PROBLEMS CONFUSE: PROSPECTIVE TEACHER STUDENTS' REASONING DIFFICULTIES

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Abstract

Difficulties in completing school mathematics material are also experienced by prospective teacher students. Proportion is the foundational mathematical material for studying other mathematics. Commonly known proportion concepts are direct and inverse proportion. Most students still have difficulty distinguishing between direct and inverse proportion. To distinguish between two problems requires proportional reasoning. Therefore, this study aimed to describe the forms of proportional reasoning difficulties of prospective teacher students in distinguishing between direct and inverse proportion. This research was a case study which conducted at a private university in Malang City, attended by 10 third-semester students. Two subjects were selected based on mathematics tests, interviews, and student willingness. The results showed that students experienced intuitive, additive, and proportion attempt difficulties in the first problem. Meanwhile, students experienced difficulties with additive and proportion attempts difficulties in the second problem. This research can provide insight into how prospective teacher students effectively teach the concept of proportion.

Keywords: *difficulty; proportional reasoning; direct proportion; inverse proportion; prospective teacher*

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INTRODUCTION

Student teachers are prepared to become educators who can contribute knowledge to students. Furthermore, they are equipped with the competencies needed to plan effective lessons (Carlson et al., 2019). The quality of their knowledge also has a significant impact on the quality of student learning (Copur-Gencturk et al., 2023; Rogers & Steele, 2016). The knowledge and skills of student teachers are essential for problem-solving and reasoning, which are essential for student teachers' competencies, particularly in mathematics (science) education, when teaching (Osborne, 2014).

Reasoning is essential in mathematics learning (Ahl, 2019; Çoban & Tezci, 2022; Parameswari et al., 2023; Rohati et al., 2021). Bronkhorst et al., (2020) define reasoning as

the ability to make claims, evaluate, and draw conclusions. Furthermore, reasoning can be described as a mental process that connects facts to draw conclusions or generalizations (Bozkuş & Ayvaz, 2018; Mata-Pereira & da Ponte, 2017). In other words, reasoning is an individual's mental process to understand existing information by connecting the information used for decision making.

Reasoning plays an important role in learning mathematics and its applications. One of the most important mathematical skills in everyday life is proportional reasoning (Beckmann & Izsák, 2015; Hilton et al., 2016; Kontogianni & Tatsis, 2019; Parameswari et al., 2023; Weiland et al., 2021). This is reinforced by the statement by Pişkin Tunç & Çakıroğlu (2022) that proportional reasoning is the most important mathematical comprehension ability because it is closely related to other fields of science and mathematics across all levels of compulsory education. Proportional reasoning refers to the ability to use ratios in situations involving proportions of quantities (Doyle et al., 2016; Hilton et al., 2016) and is the basis for understanding algebra and the transition from informal to formal mathematical thinking (Doyle et al., 2016). Proportional reasoning can also be interpreted as a person's ability to understand, construct, and use multiplicative relationships between two quantities of the same or different categories (Van Dooren et al., 2009). Proportional reasoning is often used to see the relationship between quantities. For example, if we want to estimate the amount of gasoline needed when traveling. The relationship between the amount of gasoline is related to the distance traveled. In the field of mathematics education, this proportional reasoning can be developed through the concept of proportion.

The concept of proportion (ratio and proportion) is important in school learning. Several researchers have conducted research related to the importance of the concept of proportion (Andini & Jupri, 2017; Artut & Pelen, 2015; Buforn et al., 2022; Diba & Prabawanto, 2019; Dougherty et al., 2016; Parameswari et al., 2023; Zamri, 2022). This is because proportion material forms the foundation for learning other mathematical materials (Dougherty et al., 2016; Misnasanti et al., 2017; Vanluydt et al., 2021; Weiland et al., 2021). Other mathematical materials that require the concept of proportion include algebra, geometry, statistics, and so on (Beckmann & Izsák, 2015; Misnasanti et al., 2017; Vanluydt et al., 2021). Apart from being important in learning mathematics, this concept of proportion is also useful in everyday life (Phuong & Loc, 2020).

The concepts of proportion known in schools are direct and inverse proportion. Arican & Kiymaz (2022) state that there are two types of proportional relationships between two quantities: direct and inverse proportion. This direct and inverse proportion material has been taught since elementary school (Mardika & Mahmudi, 2021; Small, 2015) and continued in junior high school (Irfan et al., 2020; Parameswari et al., 2023). Given that this material has been taught since school, students must have a strong understanding of this material before entering higher levels, especially when entering college. Therefore, prospective teacher students are considered to be able to master school material, especially material on direct and inverse proportions.

However, the fact is that many prospective teacher students still have difficulty solving proportion problems. This is reinforced by the researcher's observations when presenting inverse proportion problems to prospective teacher students at a private university in Malang City. The problems presented to the students can be seen in Figure 1 below.

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Dono, Indro, dan Kasino dapat menyelesaikan suatu pekerjaan sendirian masing-masing dalam 10 hari, 20 hari, dan 40 hari. Berapa hari yang dibutuhkan Dono untuk menyelesaikan pekerjaan tersebut jika setiap 3 hari dibantu oleh Indro dan Kasino?

Figure 1. Preliminary Study Problem

Based on the results of the preliminary study, no students were able to answer the given problem correctly. Students still had difficulty seeing the relationship between one piece of information and another. In this case, students still had difficulty determining the relationship between quantities. The researchers selected student work (S1) that was representative of other students' work. The results of the S1 work on the given problem can be seen in Figure 2 below.

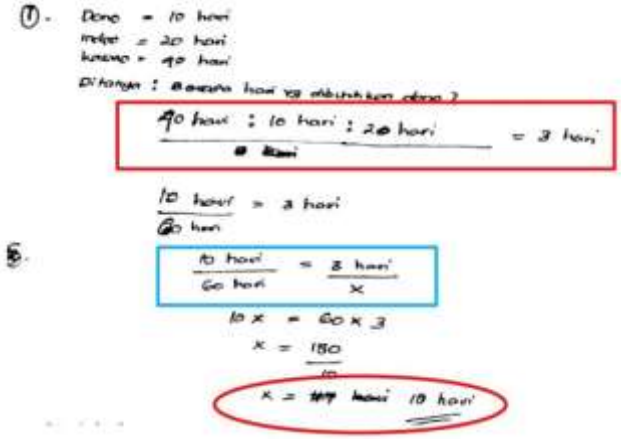

Figure 2. Results of S1 Work on Proportion Problems

Based on the student's work in Figure 2 above, it appears that S1 still struggled to solve the given problem. In the red box, S1 wrote 40 days: 10 days: 20 days = 3 days. This implies that S1 assumed that Dono, Kasino, and Indro worked together to complete the task in 3 days. This indicates that S1 was still confused about connecting the available information to the problem. Then S1 wrote that $\frac{10\ hari}{60\ hari} = \frac{3\ hari}{x}$ (blue box). He stated that if Dono, assisted by his friend, could complete the work in 3 days, then if Indro and Kasino worked together (40 days + 20 days = 60 days), the work could be completed in x days. This value of x is what S1 wants to determine. Based on the results of S1's work and interviews, the researcher suspected that S1 had difficulty connecting known information with the requested problem.

The results obtained from S1's work stated that if Dono was assisted by Indro and Kasino, the work could be completed in 10 days. While logically, the initial information stated that if Dono worked alone, the work could be completed in 10 days. So, it is impossible for Dono to be assisted by his friend to take the same time (10 days). In addition, when interviewed, S1 considered that the problem given was a problem of direct proportion. Therefore, in the process of working, S1 used the concept of direct proportion to solve the problem of inverse proportion.

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Based on the results of the observations, the researcher suspects that most students experience difficulties in proportional reasoning to distinguish between direct and inverse proportion problems. Following up on the problems that occurred, the researcher wanted to conduct a case study on students who were suspected of experiencing difficulties in proportional reasoning when distinguishing between direct and inverse proportion problems.

Several researchers have conducted research related to proportional reasoning (Ahl, 2019; Arican, 2019; Arican & Kiymaz, 2022; Buforn et al., 2022; Burgos & Godino, 2022; Cabero-Fayos et al., 2020; Copur-Gencturk et al., 2023; Copur-Gencturk, et al., 2023; Glassmeyer et al., 2021; Kontogianni & Tatsis, 2019; Parameswari et al., 2023; Pişkin Tunç & Çakıroğlu, 2022; Vanluydt et al., 2021; Weiland et al., 2021; Zamri, 2022). Ahl (2019) designed a research-based detection test to assess students' initial understanding of proportional reasoning. This task design was also carried out by Burgos & Godino (2022), where the focus of this study was on how prospective teacher elementary education students design tasks involving proportional and algebraic reasoning. Arican (2019) and Arican & Kiymaz (2022) focused on examining prospective teacher students' understanding and their ability to distinguish proportional relationships from non-proportional relationships. Meanwhile, research conducted by (Buforn et al., 2022) aimed to characterize how teachers recognize student reasoning. Then, Cabero-Fayos et al. (2020) examined the understanding and strategies used by prospective student teachers in solving inverse proportion problems using proportional reasoning. In this study, they classified several incorrect proportional reasoning strategies and correct reasoning strategies. For incorrect reasoning strategies, these consist of no answer, intuitive, additive, *proportion attempt*, and other errors. Meanwhile, correct reasoning strategies consist of *proportion formula*, proportional reasoning, algebraic, and *correct other*.

From the studies that have been reviewed previously, there has not yet examined the types of proportional reasoning difficulties of prospective teacher students when distinguishing between direct proportion problems and inverse proportion problems. Several previous studies, such as Arican (2019) and Irfan et al. (2020), have indeed addressed errors or misconceptions related to inverse proportions and the incorrect application of proportional strategies. Furthermore, that previous studies have not systematically classified types of proportional reasoning difficulties based on the framework of erroneous strategies as referenced in Cabero-Fayos et al. (2020). In this study, researchers used a guide in determining the form of proportional reasoning difficulties referring to incorrect proportional reasoning strategies based on research conducted by Cabero-Fayos et al. (2020). However, researchers only used three proportional reasoning strategies according to the results of student work consisting of: Intuitive, additive, and *proportion attempt*. Indicators of this intuitive difficulty when students cannot use appropriate information (wrong information) in determining the relationship between quantities so that students cannot distinguish or have difficulty in determining whether or not direct proportion problems. Additive difficulties experienced by students when students make mistakes in determining the focus of the given quantity relationship. *Proportion attempt* difficulties when students can explain the intended proportion but cannot explain the quantity relationship correctly.

Based on the problems above, the problem formulation of this research is "What is the form of proportional reasoning difficulties experienced by prospective teacher students in distinguish the categories of problems of direct or inverse proportion?". Therefore, this study aims to describe the forms of proportional reasoning difficulties experienced by prospective teacher students in distinguishing the categories of direct or inverse proportion problems.

https://dx.doi.org/10.26594/jmpm.v10i2.6019

METHOD

This research is a case study. This approach was chosen based on the researcher's findings regarding the forms of proportional reasoning difficulties pre-service teachers faced in distinguishing proportion problems. This is in accordance with Takona (2024) statement that case studies are useful for describing and exploring unique instances within a particular phenomenon. Case studies are specifically conducted to deepen the general public's understanding of a phenomenon (Bloomberg & Volpe, 2012). Therefore, the researcher used a mathematics test in the form of direct and mixed proportion problems as well as interview guidelines. The mathematics test was used to identify the forms of proportional reasoning difficulties of prospective teacher students. Meanwhile, the semi-structured interview aimed to confirm and deepen the students' thought processes related to proportional reasoning when distinguishing between direct and inverse proportion problems.

Researchers gave two proportion problems to 10 prospective teacher students at one of the campus in Malang City. The problems presented can be seen in Figure 3 below.

1. Mesin fotokopi P dapat mencetak sebanyak 420 lembar dalam 7 menit. Jika perbandingan kecepatan mesin fotokopi P dan Q adalah 2:3 maka berapa waktu yang diperlukan mesin fotokopi Q untuk mencetak 7200 lembar kertas?

2. Ainun mengendarai mobil dari kota Y ke kota Z dengan kecepatan 48 km/jam dan memerlukan waktu 5 jam. Mobil tersebut membutuhkan 3 liter pertalite untuk menempuh jarak sejauh 45 km. Ainun menambah kecepatan menjadi 60 km/jam. Berapa lama waktu dan berapa banyak pertalite yang diperlukan Ainun mengendarai mobil dari kota Y ke kota Z?

Figure 3. Research Problem

The results of the 10 students' work were then analyzed based on indicators of student difficulty through proportional reasoning strategies from Cabero-Fayos et al. (2020). Students who had difficulty performing proportional reasoning when distinguishing between direct and inverse ratio problems were selected as research subjects. From the results of this analysis, two research subjects were determined based on the difficulty of proportional reasoning in distinguishing between direct and inverse ratio problems in each problem given. The researcher also considered the students' fluency in communication and their willingness to be research subjects.

The researcher conducted member checking and peer discussions to obtain credible data (Nowell et al., 2017). Member checking was conducted through interviews with prospective teacher students who had difficulty distinguishing proportion problems. The researcher conducted peer discussions with fellow mathematics education lecturers outside the research team to obtain suggestions regarding the data obtained.

RESULTS AND DISCUSSION

The results of this study are in the form of a description of the form of proportional reasoning difficulties of students in distinguishing between direct proportion problems and inverse proportion problems. From the results of these tests and interviews, two research subjects were obtained, hereinafter the first subject is called S1 and the second subject is called S2. The following describes the difficulties of the research subjects when

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distinguishing between direct proportion problems and inverse proportion problems.

S1's Proportional Reasoning Difficulties in Question 1

In question 1, S1 wrote "420 = 7 *menit*" and "7200 = ?" (red box) as shown in Figure 4 below.

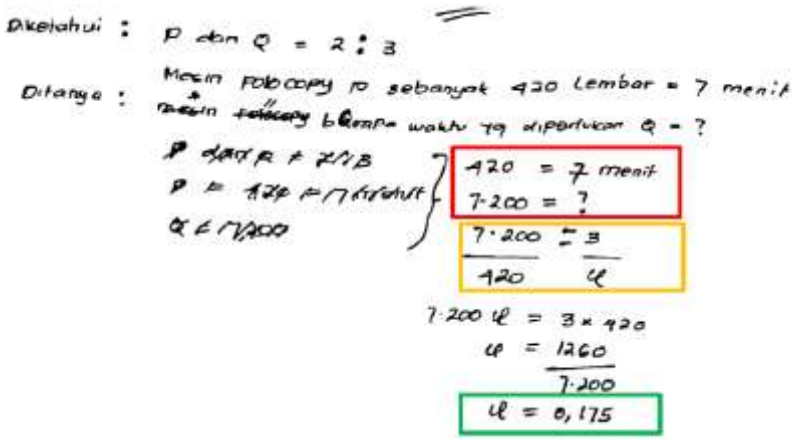

Figure 4. S1 Work on Question 1

S1 states that if photocopier P can print 420 sheets, it will take 7 minutes. So, if photocopier Q prints 7200 sheets, how long will it take? From this statement, S1 then writes: $\frac{7200}{420} = \frac{3}{x}$ (yellow box). S1 stated that the number 3 came from the information in the question which stated that the ratio of the speeds of the photocopier P and Q was 2:3. This statement by S1 indicated that S1 had incorrectly used the correct information. This error was made because S1 had difficulty connecting information about the photocopier's printing capability with the time required for the machine. This difficulty experienced by S1 was categorized as difficulty in intuitive proportional reasoning. As research conducted by Cabero-Fayos et al. (2020)found that students who made incorrect guesses or used incorrect information in solving proportion problems were categorized as having difficulty in intuitive thinking. This is also related to the meaning of intuitive thinking where if students are able to think intuitively then they are able to think that leads to ideas to solve the given problem (Baiduri et al., 2022). Therefore, students know which information should be used in solving the problem (Kurniawati et al., 2022). In other words, S1 has not been able to perform intuitive proportional reasoning.

In the process of working on question number 1, S1 was still unable to determine the focus of the problem so he made a mistake in making a relationship between two quantities, namely $\frac{7200}{420} = \frac{3}{x}$. This type of difficulty experienced by undergraduates is called additive. As defined by Cabero-Fayos et al. (2020), additive occurs when students have difficulty determining the focus of a given quantity relationship. This aligns with the opinion of (Copur-Gencturk et al., 2023) that someone who doesn't think additively is unable to accurately see the relationship between two quantities.

S1's difficulty in intuitively and additively reasoning about proportions also made it difficult for him to determine whether the problem was a proportional or inverse proportional problem. In this case, S1 assumed the problem was an inverse proportional problem. This can be seen from S1's work, where he wrote the final result as "x = 0.175." However, when interviewed by researchers, S1 stated that the time required for photocopier Q to print 7200 sheets should be longer than for photocopier P. S1 was able to state the

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expected result but incorrectly determined the quantity relationship in the problem. This type of difficulty is called a proportion attempt. Cabero-Fayos et al. (2020) state that a proportion attempt occurs when someone can explain the intended proportion but cannot correctly explain the quantity relationship.

Given the difference between S1's final result and the argument given by S1 that the time required for photocopier Q should be longer than for photocopier P, S1 struggled to distinguish whether problem number 1 was a proportional or inverse proportional problem. Due to this difficulty, S1 ultimately did not re-examine the correctness of the answers given.

S2's Proportional Reasoning Difficulties in Problem 2

Problem 2 involves the concepts of direct ratios and inverse ratios. The ability to differentiate between these two concepts can be seen in S2's ability to see the relationship or interconnection of information. The results of S2's work can be seen in Figure 5 below.

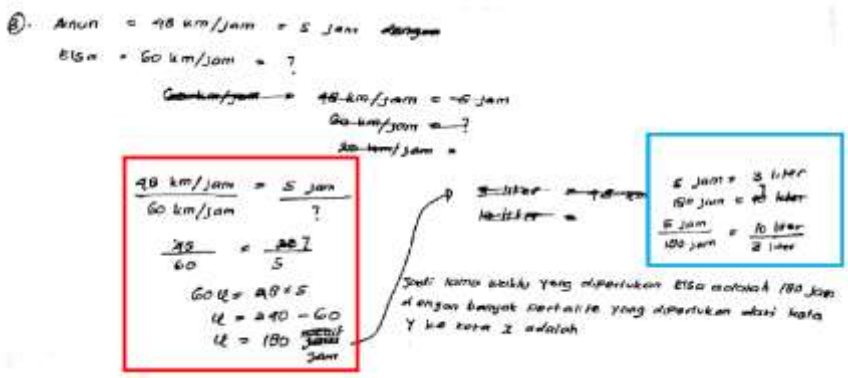

Figure 5. S2 Task for Question 2

In Figure 5 above, S2 wrote an answer to the question of how long it takes to travel from city Z to city Y at a speed of 60 km/h using the concept of inverse proportionality (red box). However, S2's result actually increases with increasing speed, namely 180 hours (red box). S2 was unable to provide a valid argument for his work. This work indicates that S2 is struggling with the proportion attempt. As is known, the proportion attempt can identify the intended proportion Cabero-Fayos et al. (2020). In this case, S2 knows that the relationship between speed and time is an inverse proportionality. However, S2 cannot explain why these two quantities must be used instead of the other.

Furthermore, S2 is unable to determine the amount of gasoline needed. S2 compares the length of the trip (travel time) with the amount of gasoline needed. S2's difficulty in determining the relationship between the amount of gasoline and the distance traveled indicates S2's difficulty with additive proportional reasoning. This is consistent with the research findings of Cabero-Fayos et al. (2020) and Copur-Gencturk et al. (2023), which found that this additive problem occurs when students incorrectly determine the focus of a given quantity relationship. Therefore, S2 students found it difficult to distinguish whether problem number 2 was a direct of proportion or an inverse proportion problems.

CONCLUSION

Based on the results and discussion reviewed, the researcher found that in distinguishing between direct proportion problems and inverse proportion problems,

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prospective teacher students experienced difficulties in proportional reasoning, namely intuitive, additive, and *proportion attempt.* In problem number 1, namely the direct proportion problem, students experienced difficulties in intuitive, additive, and *proportion attempt.* Meanwhile, for problem number 2, namely the mixed problem (direct proportion and inverse proportion), students experienced difficulties in additive and *proportion attempt.*

This intuitive difficulty occurs when students cannot use appropriate information (information which is wrong) in determining the relationship between quantities so that students cannot differentiate or have difficulty in determining whether the proportion problem is direct or not. Additive difficulties experienced by students when students make mistakes in determining the focus of the given quantity relationship. *Proportion attempt* difficulties when students can explain the intended proportion but cannot explain the quantity relationship correctly. Overall, prospective teacher students need to determine the relationships between quantities in the problem information and explain the proportions involved when distinguishing between direct and inverse proportion problems. Therefore, students often encounter difficulties with additive and proportion attempts.

The results of the study indicate that most students are unable to perform proportional reasoning effectively, while prospective teacher students must be involved in the field, teaching school mathematics to students. Therefore, follow-up or intervention is needed for students experiencing difficulties. Research is needed on how prospective teacher students teach the concept of proportion to students. Therefore, further research needs to examine the thought processes of prospective teacher students in solving proportion problems with various task designs. This study did not consider student abilities, so further research can be reviewed from the perspective of student learning styles, personality types, emotional intelligence levels, and so on.

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DECLARATIONS

Author Contribution	:	All authors contributed to conceptualizing the research idea, the theoretical review, the methodology, and the instruments. The first author was responsible for data collection and analysis, and writing the article. The second author reviewed and checked for completeness of the data. The third author validated the instruments, checked the completeness of the manuscript, and checked the references used.
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Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	The research was conducted in accordance with ethical standards. The university, lecturers, and all participants granted permission for the collection of research data.

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